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Shape of the Geomagnetic Field Solar Wind Boundary

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Abstract. The shape of the boundary of the geomagnetic field in a solar wind has been calculated by a self-consistent method in which, in first order, approximate magnetic fields are used to calculate a boundary surface. The electric currents in this boundary produce magnetic fields, which can be calculated once the first surface is known. These are added to the dipole field to give more accurate fields, which are then used to compute a new surface. This iterative procedure converges rapidly, and the final surface may be tested by finding how close the total fields outside the boundary are to the required value of zero. The result of this stringent test is that the magnetic fields in the plasma outside the fourth surface and within twice the distance to the boundary on the solar side are everywhere less than 1 per cent of the geomagnetic dipole field in the absence of a solar wind. This surface has been used to calculate the perturbation of the geomagnetic field by the solar wind; the results of these calculations, plus a number of applications, are given in an accompanying paper.

AUTHOR

INTRODUCTION

The geomagnetic dipole has been observed to be immersed in a continuous plasma stream emanating from the sun [*Neugebauer and Snyder*, 1962; *Freeman et al.*, 1963; *Bonetti et al.*, 1963]. Although small magnetic fields are present in the solar plasma [*Coleman et al.*, 1962], they only serve primarily to 'tie' different regions of the plasma together. The foremost feature of the solar plasma stream is the stream pressure it exerts on any obstruction it encounters. Such an obstruction is furnished by the geomagnetic field, which holds off the stream and creates a cavity from which the solar plasma is excluded [*Chapman and Ferraro*, 1931; *Dungey*, 1958; *Cahill and Amazeen*, 1963].

The weak interplanetary magnetic field imbedded in the plasma has little effect on the pressure conditions on the solar side of the earth. (We believe that this field is compressed against the boundary of the geomagnetic field [*Beard*, 1964*a*], and the pressure on the boundary may be essentially independent of the existence of an interplanetary field; in any case, the particle pressure will change by only a relatively constant factor, less than 2.) On the antisolar side of the earth, however, the interplanetary field becomes the predominant consideration in determining the closure of the cavity. Just how the cavity does close on the antisolar side is a very difficult problem, whose solution must await a better knowledge of the interplanetary magnetic field and the theory of plasma motion in the presence of trapped magnetic fields. Fortunately, the electric currents existing on the boundary on the antisolar side are very weak and relatively distant from the earth's surface, compared with the solar-side boundary. Therefore, our understanding of the solar-side-boundary shape and the distortion of the geomagnetic field does not depend very much on whether an interplanetary field is included in our considerations, and so there appears to be adequate justification for simplifying the problem by assuming the solar wind to be free from magnetic field and at zero temperature (no lateral motion of the ions perpendicular to the stream velocity occurs).

Several attempts have been made recently to calculate the shape of the boundary under the assumptions of a field-free, zero-temperature plasma specularly reflected from the boundary [*Beard*, 1960; *Beard*, 1962; *Beard and Jenkins*,

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1962; *Spreiter and Briggs,* 1962; *Midgley and Davis,* 1963]. All have assumed certain approximations that limit their accuracy. The best of them is the solution presented by Midgley and Davis, which will be compared with ours below. *Slutz* [1963] has obtained a solution to a problem with somewhat different boundary conditions at the interaction surface. *Beard* [1964*b*] has recently reviewed all the calculations made to date. Other recent reviews of the solar wind–earth interaction have been written by *Chapman* [1963, pp. 371, 421], *Hines* [1963], and *Blum* [1963].

A new calculation of the boundary of the geomagnetic field, using a self-consistent method, is presented in this paper. The self-consistent method has been introduced previously [*Beard,* 1962; *Beard and Jenkins,* 1962] to illustrate the convergence of the particular approximation to the boundary conditions ordinarily used in the calculation. The accuracy and convergence of the self-consistent method have been illustrated for a variety of simple problems by *Baker et al.* [1964], who have found that the second- or third-order approximation is indistinguishable from the exact solution of problems for which analytic solutions exist.

The method consists of using an approximate magnetic field in the differential equations representing the boundary conditions, from which a first-approximation boundary surface may be computed. From the surface currents on this first surface a correction field due to the surface curvature may then be determined which is added to the dipole field to give a better approximation to the magnetic field just inside the boundary. This more accurate field is then used to determine a second surface, and the process is continued. It is found that, by the time the third surface is calculated by this method, negligible improvement over the previous surface is obtained. Magnetic fields due to the surface currents computed within the plasma, outside the magnetosphere, are almost everywhere found to be within a few parts in a thousand of canceling the geomagnetic dipole field alone at each point, and thus the plasma is essentially field-free, as required by the boundary conditions. Such excellent cancellation of the field within the plasma leads us to believe that the calculation of the geomagnetic field boundary presented here is everywhere accurate to less than 1 per cent, within the assumptions made about the model and the interplanetary medium.

Method of Calculation of the Boundary

Basic equations. With the assumptions of a zero-temperature, field-free plasma incident perpendicular to the geomagnetic dipole and undergoing specular reflection, the boundary shape may be determined by equating the stream pressure of the ions on one side of the boundary to the magnetic field pressure on the other side [*Chapman and Ferraro,* 1931; *Ferraro,* 1952; *Spitzer,* 1956; *Dungey,* 1958; *Beard,* 1960]:

$$p = 2nmv^2 \cos^2 \psi = B_t^2/8\pi \tag{1}$$

where n is the ion density in the solar wind, m is the ion mass, v is the stream velocity, ψ is the angle between the outward vector normal to the boundary and the stream velocity vector, and B_t is the total magnetic field intensity just inside the boundary. If $\hat{\mathbf{n}}_s$ represents the unit outward vector normal to the surface, $\cos \psi = \hat{\mathbf{n}}_s \cdot \hat{\mathbf{v}}$, $\mathbf{B}_t$ must be tangential to the surface, and (1) may be rewritten [*Davis and Beard,* 1962]

$$|\hat{\mathbf{n}}_s \times \mathbf{B}_t| = -(16\pi nmv^2)^{1/2}\hat{\mathbf{n}}_s \cdot \hat{\mathbf{v}} \tag{2}$$

where $\hat{\mathbf{v}}$ is the unit vector in the direction of the stream velocity. The minus sign is necessary when the square root is taken, since ψ must always be greater than 90° in order that the boundary surface may be everywhere exposed to the stream pressure.

Equations 1 and 2 are equivalent only if the total field $\mathbf{B}_t$ is really tangential to the boundary. The approximation for $\mathbf{B}_t$ used to calculate a first surface is definitely *not* tangential. However, one test of the self-consistent method is to determine the extent to which the modified $\mathbf{B}_t$ calculated in higher approximations is actually tangential to the surface. The results of this test will be given in the section on results.

The total magnetic field just inside the boundary, $\mathbf{B}_t$, may be considered to be composed of three parts: (1) the geomagnetic dipole field at that point, $\mathbf{B}_g$; (2) a surface current planor field, $\mathbf{B}_p$, equal to the field produced by an infinite plane sheet of current tangential to the surface; and (3) a surface current field resulting from the curvature of the surface, $\mathbf{B}_c$, calculated at a position at the center of the surface currents. Thus the field just inside the surface due to the

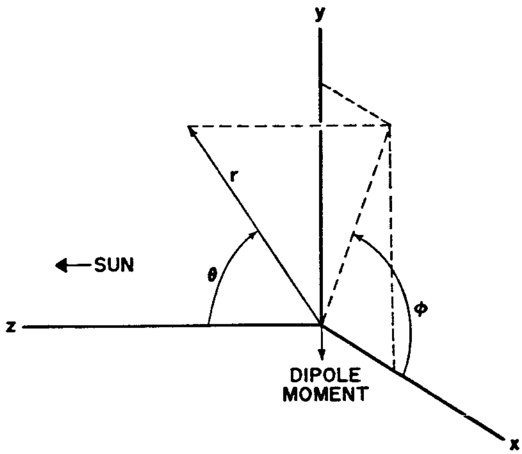

Fig. 1. Coordinate system used in the calculation. Note that the polar axis points toward the sun, instead of the north star.

surface currents is $\mathbf{B}_p + \mathbf{B}_c$, and the total field at that point is

$$\mathbf{B}_t = \mathbf{B}_g + \mathbf{B}_p + \mathbf{B}_c \tag{3}$$

If we assume that the boundary is infinitesimally thin, and move to a position just outside the boundary, $\mathbf{B}_g$ and $\mathbf{B}_c$ are unchanged but $\mathbf{B}_p$ changes sign, and

$$\mathbf{B}_{\text{outside}} = \mathbf{B}_g - \mathbf{B}_p + \mathbf{B}_c = 0 \tag{4}$$

since it has been assumed that the plasma is field-free. Thus

$$\mathbf{B}_p = \mathbf{B}_g + \mathbf{B}_c \tag{5}$$

$$\mathbf{B}_t = 2(\mathbf{B}_g + \mathbf{B}_c) \tag{6}$$

and (2) becomes

$$|\hat{\mathbf{n}}_s \times (\mathbf{B}_g + \mathbf{B}_c)| = -(4\pi n m v^2)^{1/2}\hat{\mathbf{n}}_s\cdot\hat{\mathbf{v}} \tag{2'}$$

where thus far no approximations have been made.

Calculation of the first-approximation boundary surface. The curvature field $\mathbf{B}_c$ cannot be determined until we have a first surface, but the first surface cannot be obtained without knowing the field. A reasonable first approximation consists of ignoring the curvature field and solving (2′) for $\mathbf{B}_c = 0$. This is equivalent to assuming that the field just inside the surface is equal to twice the component of the geomagnetic field tangential to the boundary [*Beard*, 1960; *Spreiter and Briggs*, 1962]

$$|\mathbf{B}_t| \approx 2\,|\hat{\mathbf{n}}_s \times \mathbf{B}_g| \tag{7}$$

The coordinate system used in the calculation is illustrated in Figure 1. Spherical coordinates are used, but, instead of the usual orientation, the polar or z axis is antiparallel to the solar plasma stream, the y axis points north, and the angle ϕ is measured from the magnetic equatorial plane. In this coordinate system

$$\hat{\mathbf{n}}_s = a\left(\hat{\mathbf{r}} - \frac{1}{r}\frac{\partial R}{\partial \theta}\hat{\boldsymbol{\theta}} - \frac{1}{r\sin\theta}\frac{\partial R}{\partial \phi}\hat{\boldsymbol{\phi}}\right) \tag{8}$$

$$\hat{\mathbf{v}} = -\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\boldsymbol{\theta}} \tag{9}$$

$$\mathbf{B}_g = (M/r^3)(-2\sin\theta\sin\phi\hat{\mathbf{r}} + \cos\theta\sin\phi\hat{\boldsymbol{\theta}} + \cos\phi\hat{\boldsymbol{\phi}}) \tag{10}$$

where the surface is defined by $r = R(\theta, \phi)$, $\mathbf{B}_g$ is the geomagnetic dipole field with magnetic moment $\mathbf{M}$, and a is a normalizing factor that makes $\hat{\mathbf{n}}_s$ a unit vector. Since $\hat{\mathbf{n}}_s$ appears on both sides of (2′), this factor cancels out of the equation.

In performing the calculations, all distances are expressed in units of

$$r_0 \equiv (M^2/4\pi n m v^2)^{1/6} \tag{11}$$

This is a convenient unit of distance, since it represents the altitude of the subsolar point in the first approximation surface, where the field at that point is assumed to be twice the dipole field. When r is expressed in these units, the constant factor under the square root sign in (2′) is removed, which makes all quantities dimensionless. $\mathbf{B}_c$ is then in units of M/r_0^3.

When equations 8, 9, and 10 are substituted into 2′, a complicated nonlinear, partial differential equation in two unknowns, θ and ϕ, results. A Newton-Raphson technique was used with the aid of an IBM 7094 computer to solve this equation for the entire three-dimensional surface. The surface was divided into a grid of 5° increments in θ and ϕ. It was necessary to determine only a fourth of the entire surface, since the remainder is completely symmetric. The solution starts at the subsolar point, where, because of symmetry, $\partial R/\partial\theta$ and $\partial R/\partial\phi$ are zero. With this simplification, (2′) can be directly solved for r at the subsolar point, giving

$$r_{ss} = (1 - B_{c\phi})^{-1/3} \tag{12}$$

in dimensionless units. For the first surface $\mathbf{B}_c = 0$, and therefore $r_{ss} = r_0$ as indicated earlier.

To obtain the complete surface, the computer moves out from the subsolar point, first along the equator ($\phi = 0°$) in 5° increments of θ. On the equator $\partial R/\partial\phi = 0$ because of symmetry. At each point the computer makes a first guess at a value of r. Then $\partial R/\partial\theta$ is determined by fitting a parabola to this point plus the previous two computed points and finding the slope. These values of r and $\partial R/\partial\theta$ are substituted into (2′), and the difference between the two sides of the equation is computed. In general, this difference will not be precisely zero, since the first guess was not exactly correct. A small variation is made in r, and the new values of r and $\partial R/\partial\theta$ are used to calculate a new difference. If r_1 and r_2 are the two trial values, d_1 and d_2 are the resulting differences, and the variations are small enough so that the differences depend linearly on r in this region, then the 'correct' value is

$$r_3 = (d_2r_1 - d_1r_2)/(d_2 - d_1) \tag{13}$$

In practice, r_3 is used as a new guess, another variation is made, smaller than the preceding one, and the process is repeated. The procedure continues until r is obtained to the desired degree of accuracy. In the present case an accuracy of 10^{-7} was required. Since the method is rapidly convergent, this accuracy was achieved with only two or three variations at each point.

After points on the equator are mapped out, ϕ is increased by 5° and the process is continued. Now $\partial R/\partial\phi$ is no longer zero and must also be determined at each point in the same fashion as $\partial R/\partial\theta$ by using the trial value of r plus the points immediately adjacent in ϕ. By this process, the entire three-dimensional surface can be mapped out with great accuracy. The computer takes only about 15 seconds to generate a complete surface. The basic subroutine that keeps track of all the variations was dubbed Maze, because of the multitude of decisions which it has to make.

Calculation of the curvature field. The next step in the self-consistent method is to use this first surface to compute $\mathbf{B}_c$, the additional magnetic field due to the curvature of the surface [*Beard*, 1962]. This is done by integrating over all the surface currents, using the Biot-Savart law

$$\mathbf{B}_c(\mathbf{r}) = \int \frac{\mathbf{J} \times \mathbf{r}'}{|\mathbf{r}'|^3}\, dS \tag{14}$$

where $\mathbf{J}$ is the surface current per unit length of surface (emu/sec/cm) and $\mathbf{r}'$ is the vector from the differential surface element dS to the point $\mathbf{r}$, which is taken to be at the center of the surface currents. This curvature field would be identically zero if the surface were a plane.

The direction and magnitude of $\mathbf{J}$ are easily obtained, since the boundary is infinitesimally thin and the current must produce a field discontinuity $B_t = 4\pi J$. Since $\mathbf{B}_t$, $\mathbf{J}$, and $\hat{\mathbf{n}}_s$ must be mutually perpendicular, (2) and (11) yield

$$\mathbf{J} = \frac{1}{4\pi}\,\hat{\mathbf{n}}_s \times \mathbf{B}_t = \frac{1}{2\pi}\,\hat{\mathbf{n}}_s \times (\mathbf{B}_g + \mathbf{B}_c)$$
$$= \frac{M}{2\pi r_0^{\,3}} \cos\psi\,\hat{\jmath} \tag{15}$$

where $\hat{\jmath}$ is a unit vector in the direction of $\mathbf{n}_s \times \mathbf{B}_t$. For purposes of machine computation, the expression used for the curvature field was

$$\mathbf{B}_c(\mathbf{r}) = \frac{1}{2\pi}\int \frac{[\hat{\mathbf{n}}_s \times (\mathbf{B}_g + \mathbf{B}_c)] \times \mathbf{r}'}{|\mathbf{r}'|^3}\, dS \tag{16}$$

where the value of $(\mathbf{B}_g + \mathbf{B}_c)$ is the previous approximation value taken at the position of the surface element dS. Thus $\mathbf{B}_c(\mathbf{r}')$ in the integrand of (16) is assumed zero in computing $\mathbf{B}_c(\mathbf{r})$ for the second-order approximation.

The machine calculation of $\mathbf{B}_c$ must be performed with care, since a local singularity exists in (16) as $\mathbf{r}' \to 0$. This is a finite singularity, however, since, as the surface in the neighborhood of $\mathbf{r}$ approaches a plane, contributions to the curvature field from symmetric surface elements on opposite sides of $\mathbf{r}$ tend to cancel each other. A 5° mesh size was used in the numerical integration, and contributions to the integral were included from every point up to within 5° of θ and ϕ. The region within 5° was excluded. This resulted in an estimated error of less than 5 per cent in computing the curvature field. An additional inaccuracy exists in regions near the polar axis of the coordinate system (i.e., near the subsolar point), because the grid in these regions is not very rectangular; therefore the cancellations near the singularity are not as complete. At the subsolar point itself the symmetry is restored, and the value of $\mathbf{B}_c$ calculated here is accurate. The values of $\mathbf{B}_c$ in the region from

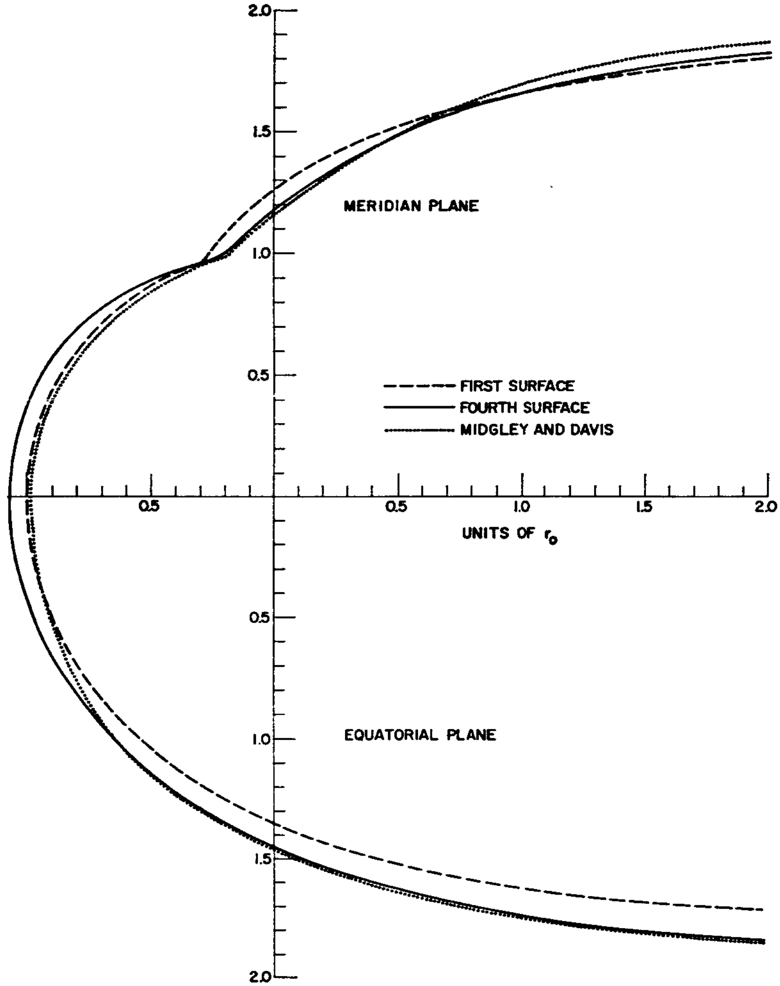

Fig. 2. Comparison of the first-approximation surface, the fourth-approximation surface, and the surface obtained by *Midgley and Davis* [1963], in the meridian and equatorial planes.

$\theta = 5°$ to $\theta = 35°$ were therefore not calculated directly, but quadratic interpolations were made for the three components from $\theta = 0°$ to $\theta = 40°$, where the grid was reasonably rectangular. Since the calculation of (16) was quite lengthy, computation time was shortened by computing $\mathbf{B}_c$ for every 10° in θ and every 15° in ϕ, and then quadratically interpolating to obtain values at intermediate points. In this way the process of determining the curvature field over the whole surface took about 6 minutes on the IBM 7094.

A second surface can now be calculated from (2′) by using the curvature fields from the first surface. A new set of curvature fields is then computed, and this process is continued until the changes from one surface to the next are negligible. One test of the convergence of this method is to see how much $\mathbf{B}_c$ changes from one surface to the next. It was found that the curvature fields calculated from the first surface averaged about 20 per cent of the dipole field in the equatorial regions but were actually larger than the dipole field in many regions near the poles. The curvature fields from the second surface were about 5 per cent different, on the average, from those based on the first surface. The third-surface fields changed by 1 or 2 per cent in the equatorial regions and about 5 per

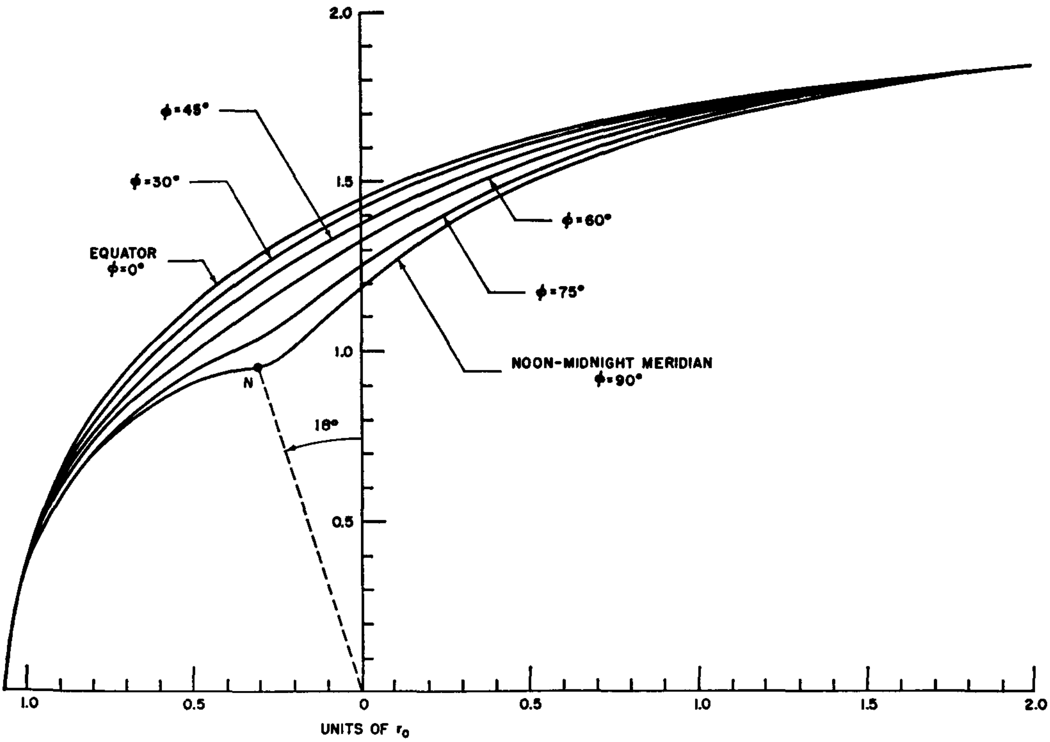

Fig. 3. Intersection of the fourth-approximation surface with planes of constant ϕ. The surface at $\phi = 15°$ is not shown, since it is almost indistinguishable from the equatorial surface. N is the position of the null point in the noon meridian.

cent near the poles. After this, the changes in the curvature fields were almost everywhere less than 1 per cent.

Results

The intersection of the first-approximation surface with the equatorial plane and with the noon-midnight meridian plane is shown as a dashed line in Figure 2. The trace in the equatorial plane is identical to that calculated by *Beard* [1960]. The trace in the meridian plane is also the same (r = constant = r_0) to within about 20° of the pole on the day side. Past that point, it corresponds essentially to the solution given by Beard, in his equations 27 and 38, but using the integration constant of *Spreiter and Briggs* [1962]. The Newton-Raphson method could not give a complete first surface in the meridian plane ($\phi = 90°$), where $\partial R/\partial\phi = 0$ because of symmetry. The surface shown is actually a smooth continuation of the surface determined at adjacent values of $\phi = 80°$ and $\phi = 85°$, which the computer had no difficulty in calculating. In higher orders, when the curvature fields are used in the meridian plane, the difficulty in determining a surface is no longer present, although the shape of the surface changes rapidly in the vicinity of the null point.

A second surface is generated by calculating the curvature fields from the first surface and using them in (2′). This surface is about 7 per cent farther out everywhere in the equatorial plane. In the meridian plane, the second surface is outside the first surface near the equator, inside near the pole, and slightly outside again in the distant antisolar regions.

The third and fourth surfaces differ from the second by less than 0.5 per cent everywhere near the equator and therefore are graphically indistinguishable from the second surface in this region. In the polar regions the differences are somewhat greater, the fourth surface differing from the third by as much as 1 per cent. A fifth surface has been calculated; it is intermediate between the third and the fourth. Every indication suggests that the process converges very rapidly, and that any number of additional surfaces could be generated with essentially no further change. The process was not carried further than a fifth surface, because the change in the curvature fields from surface to surface was already much less than the accuracy with which these fields are calculated.

The effect of the higher-order approximations

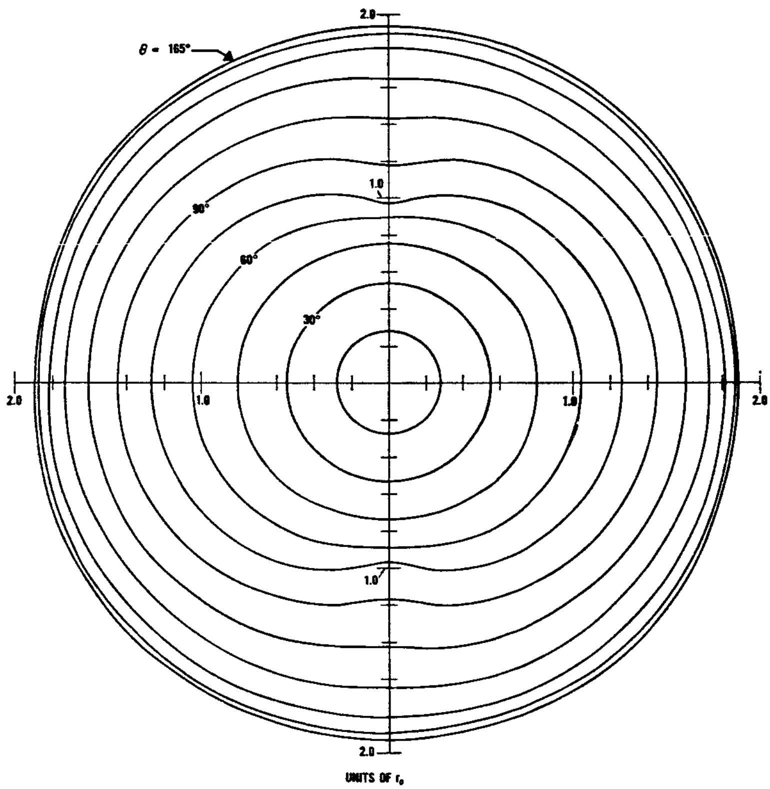

Fig. 4. A view of the magnetopause from the position of the sun. Each curve represents a cross section of the surface at constant θ. The surface becomes almost cylindrical in the distant antisolar regions.

TABLE 1. Coordinates of the Fourth Surface in Units of r_0 (15° intervals)
ϕ = 0° represents the equator, and ϕ = 90° is the noon-midnight meridian.

ϕ =	0°	15°	30°	45°	60°	75°	90°
θ = 0°	1.068	1.068	1.068	1.068	1.068	1.068	1.068
15°	1.076	1.075	1.074	1.071	1.069	1.068	1.067
30°	1.099	1.096	1.090	1.081	1.072	1.065	1.062
45°	1.142	1.136	1.123	1.103	1.079	1.059	1.051
60°	1.209	1.201	1.180	1.146	1.102	1.054	1.028
75°	1.307	1.297	1.272	1.229	1.168	1.088	1.001
90°	1.450	1.440	1.416	1.375	1.319	1.245	1.172
105°	1.663	1.656	1.638	1.610	1.571	1.522	1.479
120°	1.998	1.994	1.989	1.978	1.959	1.928	1.898
135°	2.574	2.576	2.586	2.595	2.594	2.574	2.549
150°	3.747	3.757	3.788	3.820	3.838	3.824	3.795
165°	7.304	7.330	7.405	7.482	7.529	7.512	7.461

TABLE 2. Coordinates of the Surface near the Null Point (5° intervals)

$\varphi =$	70°	75°	80°	85°	90°
$\theta =$ 60°	1.069	1.054	1.041	1.031	1.028
65°	1.076	1.057	1.037	1.021	1.016
70°	1.091	1.066	1.039	1.012	1.003
75°	1.116	1.088	1.054	1.014	1.001
80°	1.154	1.125	1.090	1.046	1.032
85°	1.206	1.178	1.145	1.105	1.091
90°	1.271	1.245	1.217	1.183	1.172

can be seen in Figure 2, which shows the fourth surface plotted along with the first surface in the two planes of symmetry, $\phi = 0°$ (equator) and $\phi = 90°$ (meridian). Figure 3 shows the complete three-dimensional fourth surface, giving the shape at intermediate values of ϕ. Figure 4 shows the cross section of the surface at constant values of θ, i.e., the approximate intersection of the surface with planes perpendicular to the earth-sun line. Here the behavior near the null point can be clearly seen. At very large distances on the dark side, near $\theta = 180°$, the cross section is almost circular, with a meridional diameter of $3.88r_0$ and an equatorial diameter of $3.78r_0$. Thus, if the solar wind were truly field-free and at zero temperature, and if the boundary were at 10 earth radii at the subsolar point, where $r = 1.068r_0$, the diameter of the approximately cylindrical surface cross section at large distances on the dark side would be about 36 earth radii.

The null point in the meridian plane is at a geomagnetic latitude of 72°, i.e., 18° from the pole, and on the sunward side. The coordinates of the fourth surface are given in Tables 1 and 2.

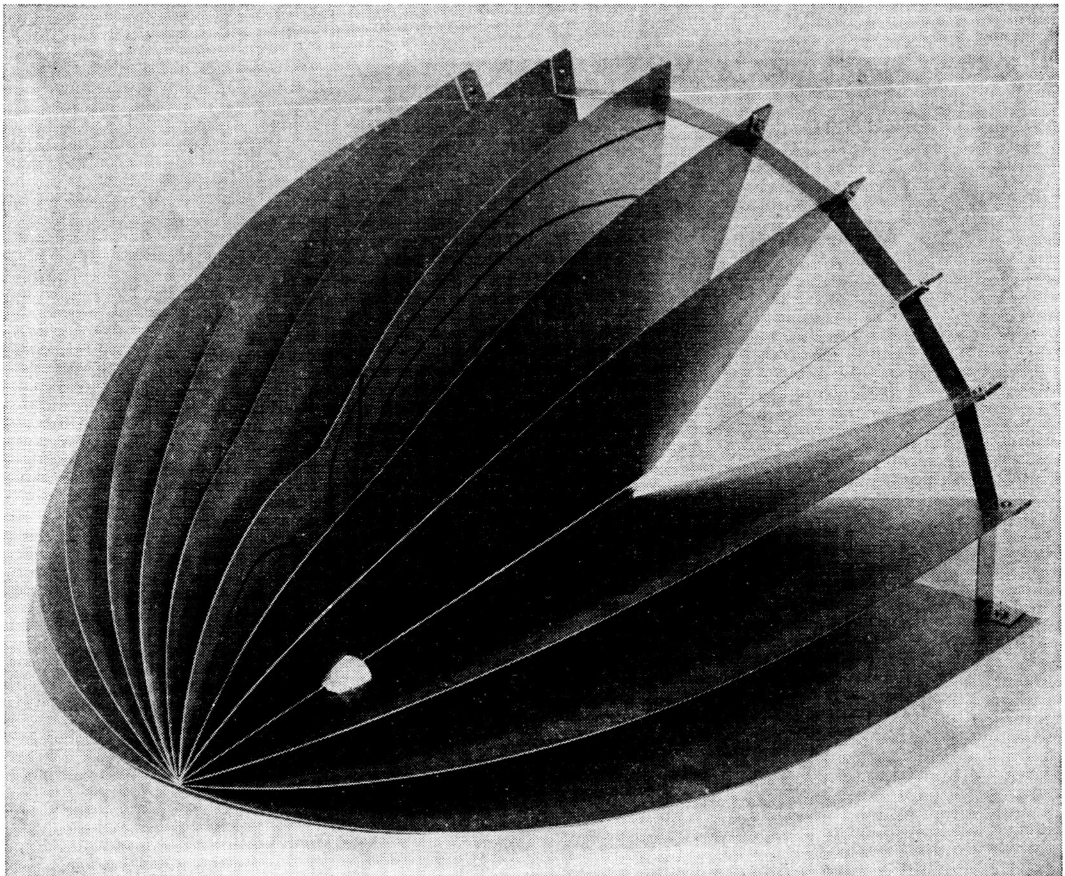

Fig. 5. A model of the magnetopause, based on the fourth-approximation surface. Some of the field lines in the noon-midnight meridian plane are shown.

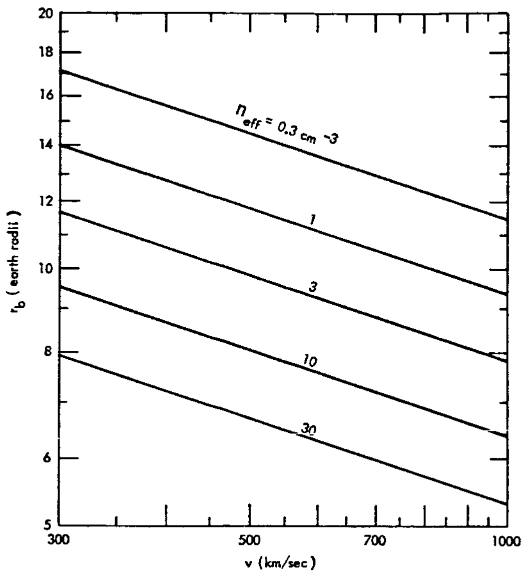

Fig. 6. The distance to the boundary along the earth-sun line as a function of solar wind parameters. v is the plasma velocity, and $n_{eff} = n_{proton} + 4\ n_{alpha}$ is the effective ion density.

A model was constructed in order to visualize the complete surface. Templates of the surface for every 15° interval in ϕ were used to cut out aluminum sections, and these sections were joined together as shown in Figure 5. A small sphere with radius $r_0/10$ was placed at the position of the earth. A few of the calculated field lines in the noon-midnight meridian are also shown.

In the present analysis, the solar wind intensity affects the size but not the shape of the magnetopause. Once the distance to the boundary along the earth-sun line is fixed, all other dimensions are determined. This distance, r_b, given by 1.068 $(M^2/4\pi\ nmv^2)^{1/6}$, is plotted in Figure 6 as a function of solar wind parameters.

A very stringent test of any proposed plasma-magnetic field boundary, given by *Midgley and Davis* [1963], is to compute the magnetic field at various points outside the magnetosphere as the sum of the field due to surface currents plus the dipole field, and compare this field with the undisturbed dipole field at each point. We have computed the field at $r = 1.5r_0$ and $2.0r_0$ in intervals of 15° everywhere on the solar side of the surface. We find that *the ratio of the computed total field in the plasma to the dipole field at every point is less than 1 per cent*, usually a few tenths of a per cent. Farther out from the surface the surface current field falls off as $1/r^3$, and at $r = 10r_0$ the total field is everywhere less than 3 per cent of the dipole field. It is believed that better accuracy could be obtained by improving the calculation of $\mathbf{B}_c$, but this is not justified in view of the changes introduced by the presence of a weak interplanetary magnetic field and time fluctuations in the solar wind pressure.

Another test that can be made is to determine the extent to which the total computed field just inside the boundary, $\mathbf{B}_t$, is tangential to the surface at every point. This was done in two ways. First, $|\hat{\mathbf{n}}_s \times \mathbf{B}_t|/|\mathbf{B}_t|$, i.e., the ratio of the component of the field tangential to the surface to the total field, was calculated at each point. This ratio must be unity if (1) and (2) are really equivalent. Over most of the surface this ratio was greater than 99.9 per cent; it was greater than 99 per cent everywhere except within 20° of the null point and within 30° of the antisolar point. It was greater than 90 per cent to within 10° of the null point and the antisolar point. At these two points, of course, the total field is zero and the ratio is indeterminate.

Second, the final surface was recomputed using $|\mathbf{B}_t|$ instead of $|\mathbf{n}_s \times \mathbf{B}_t|$ to represent the magnetic pressure. The surfaces differed by less than 1 part in 1000 except near the null point and the antisolar point, where changes of as much as 1 or 2 per cent were noted.

The fourth surface was compared with the calculation recently published by *Midgley and Davis* [1963]. In order to plot on the same scale, their basic unit of length, R_n, must be compared with r_0. From their equation 5.3

$$R_N^3 = -(M/S_{11}^1)(\pi/mnv^2)^{1/2} \qquad (17)$$

with $S_{11}^1 = -7.0030$, whereas from our equation 11

$$r_0^3 = (M/2\pi)(\pi/mnv^2)^{1/2} \qquad (18)$$

so that

$$R_N = (2\pi/7.0030)^{1/3} r_0 = 0.9645 r_0 \qquad (19)$$

The tabular values for Midgley and Davis' 'smoothed' surface were multiplied by the above factor and plotted in Figure 2 as a dotted line. It is seen that their surface is actually slightly *inside* even the first-approximation surface in the region of the subsolar point, as they have also noted. The fraction of the field just inside the surface contributed by the dipole field can be obtained from (2) and (11). At the subsolar point

$$B_t = (16\pi n m v^2)^{1/2} = 2M/r_0^3 \qquad (20)$$

and thus

$$\frac{B_g}{B_t} = \frac{M/r_{ss}^3}{2M/r_0^3} = \frac{1}{2(r_{ss}/r_0)^3} \qquad (21)$$

From Midgley and Davis' surface, $r_{ss} = 0.981r_0$, and therefore 53 per cent of the field is contributed by the dipole. For our surface, $r_{ss} = 1.068r_0$ and 41 per cent of the field is contributed by the dipole. With a plane or spherical boundary, the exact figures are 50 and 33 per cent, respectively. We feel certain that the dipole must produce less than half of the total field just inside the boundary, and that therefore the true surface must be larger than r_0 at that point. In other words, the curvature field $\mathbf{B}_c$ *adds* to the dipole field at the subsolar point rather than opposes it.

Midgley and Davis' surface moves out more rapidly than our fourth surface away from the subsolar point, and for all values of θ greater than about 60° the two surfaces are very close together. Midgley and Davis determine the null point to be 15° from the pole as opposed to 18° for our surface.

Conclusions

The surface boundary between the solar wind and the geomagnetic field has been calculated by a self-consistent method in which, in first approximation, the magnetic field resulting from the electric currents on the boundary has been approximated by treating the radius of curvature of the boundary surface as infinite. In succeeding approximations the magnetic field resulting from the currents on the boundary is computed by integration over the preceding surface and used to calculate a new surface. The method converges rapidly, with an ultimate precision limited by the precision with which the magnetic fields are computed. The third and fourth surfaces are almost indistinguishable from the second. The first and fourth approximations have been compared with Midgley and Davis' results and are illustrated in Figure 2. The final result, the fourth-approximation surface, is shown in more detail in Figures 3 and 4. A model of the surface is pictured in Figure 5. The accuracy of the fourth-approximation surface may be tested by computing the magnetic field outside the boundary (inside the plasma), which should be zero in the absence of an interplanetary magnetic field. The ratio of this computed field to the geomagnetic dipole field out to $r = 2r_0$ in the plasma region is everywhere less than 1 per cent and of the order of 0.1 per cent in most cases. The computed field just inside the surface used to balance the particle pressure is found to be tangential to the surface except near the null point and in the distant antisolar regions.

These last tests are such stringent requirements on the solution and are so well satisfied by the final results that we believe this solution to the theoretical problem (a magnetic dipole in a magnetic-free monodirectional plasma stream) to be as accurate a test of the reliability of the calculational method and as accurate a proposed geomagnetic field boundary as could be desired in the light of actual geophysical conditions.

In view of the overwhelming forward pressure of the solar wind, modifications in the surface calculation introduced by weak interplanetary magnetic fields will be negligible on the part of the surface where the stream pressure is appreciable ($\mathbf{n}_s \cdot \mathbf{v} \sim 1$, the solar side); moreover, only minor modifications of the magnetic field in the region of the earth computed from these surface currents may be expected in view of the unchanged boundary position on the solar side, where the surface currents are large and close to the earth. Introduction of these higher-order effects into the calculations is difficult at present because of our lack of experimental knowledge of the interplanetary magnetic field and lack of theoretical knowledge of the behavior of a high-velocity plasma stream containing a trapped magnetic field when encountering an obstacle to its flow. Although pressure balance conditions will remain unchanged on the solar side, the antisolar shape of the boundary will be greatly affected and intuitively may be expected to close in a long raindrop-shaped tail rather than remain open as in the problem considered in this paper. These alterations in the position of very weak and very distant surface currents should have only a negligible effect on the distortion of the geomagnetic field of the earth within about 10 earth radii of the surface of the earth. Thus the present calculation furnishes an excellent basis for computation of the magnetic field within the magnetosphere. The results, along with a number of applications, are presented in an accompanying paper [*Mead*, 1964].

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